

# **Efficient deterministic inversion of geophysical data constrained by Deep Generative Models to estimate subsurface parameters with defined spatial patterns**

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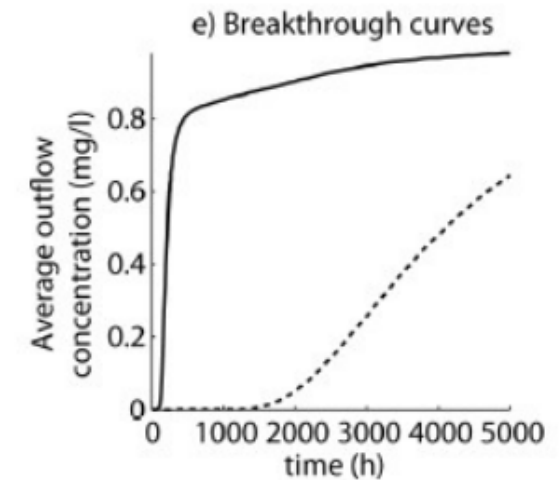
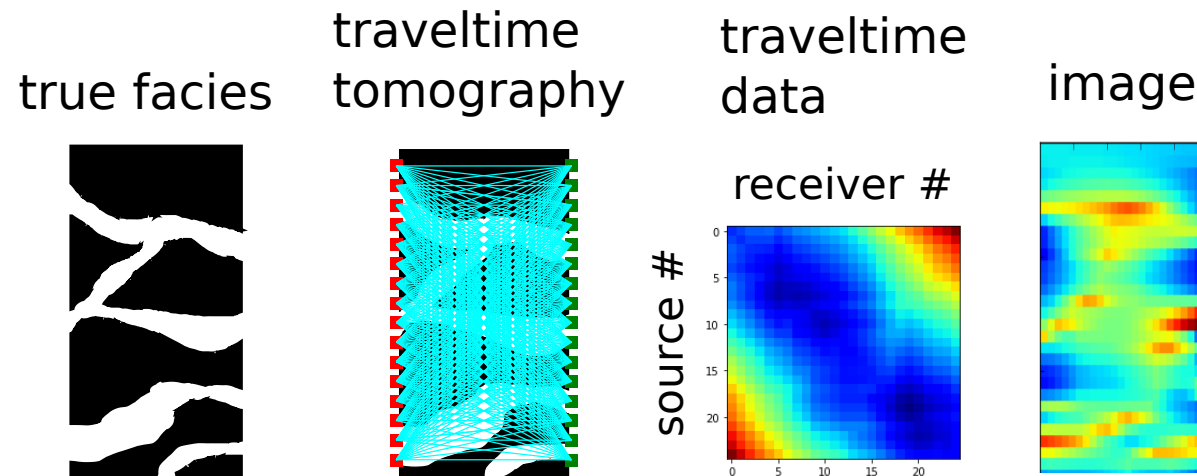
AGU Fall Meeting, December 2019



## Background

Classical imaging algorithms sometimes fail in reproducing realistic structures.

This may in turn result in wrong predictions.

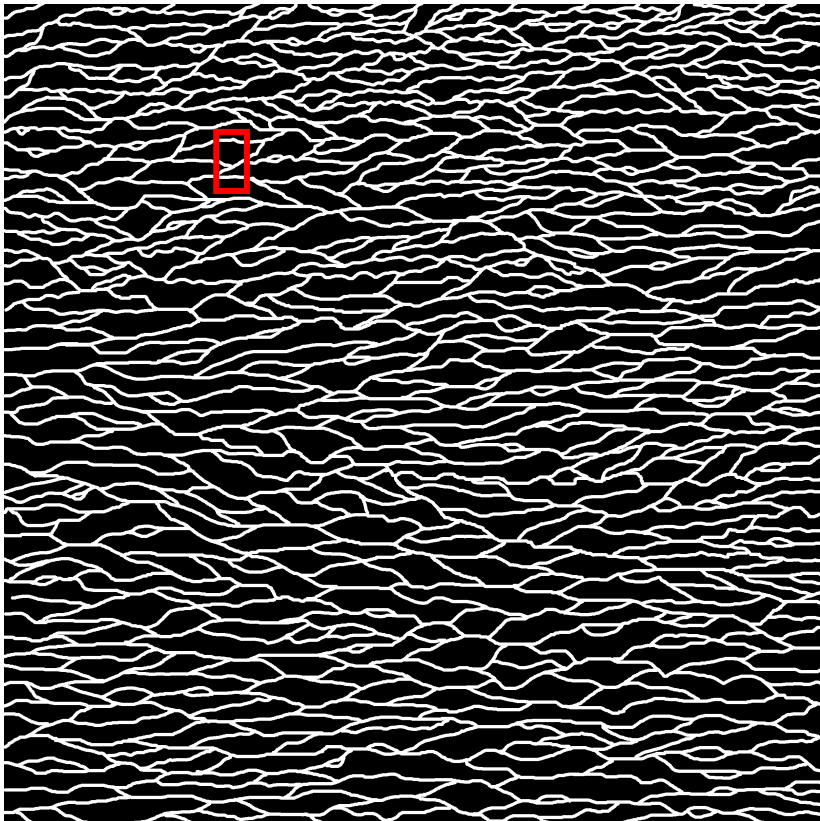


**Solution:** constrain the inversion to display the expected patterns (e.g. multiple-point statistics).

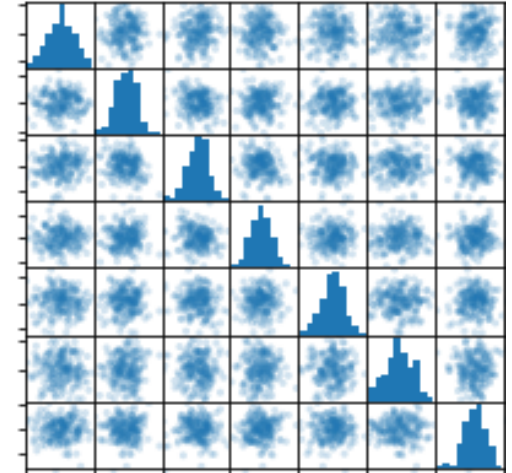
**Difficulty:** high-dimensional parameters usually require large number of samples (simulations) for inference.

## Deep Generative Models (DGM)

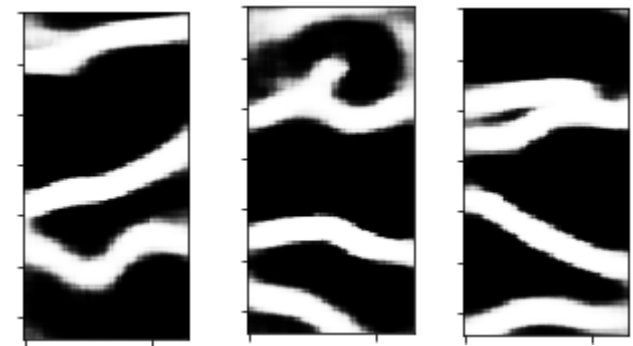
- 1 Train the DGM with many realizations (images) of expected patterns.



- 2 A latent space with lower number of dimensions is created.

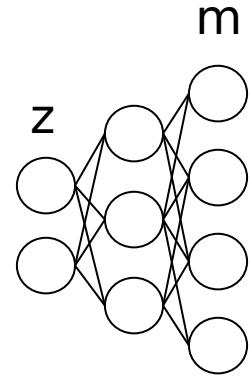


- 3 Samples with the patterns are obtained by sampling in the latent space and passing through a deep neural network.



# Deep Generative Models (DGM)

Implicit generative modeling



Two big contenders: VAE and GAN

## Variational Autoencoder (VAE)

Training: KL-divergence and reconstruction loss.

Advantages: easy training and high diversity.

Issues: oversmooth samples.

## Generative Adversarial Network (GAN)

Training: Adversarial loss (generator vs discriminator).

Advantages: sharp samples.

Issues: difficult training and mode collapse (lack of diversity).

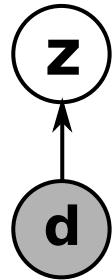
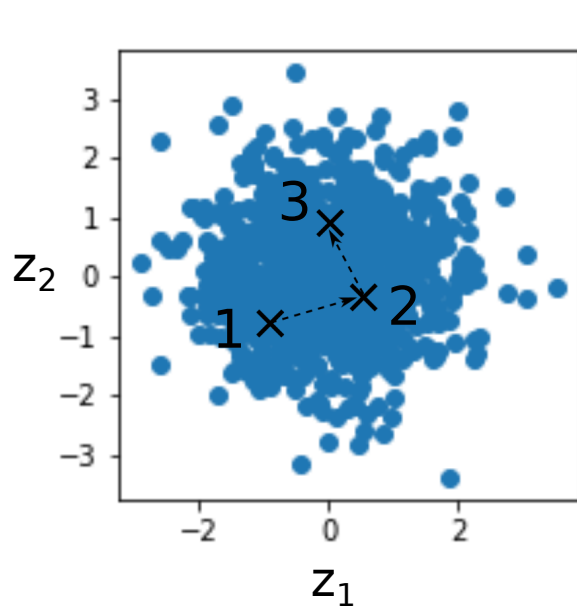
# Optimization of the objective function

**Global optimization** - computationally expensive

## Gradient-based optimization

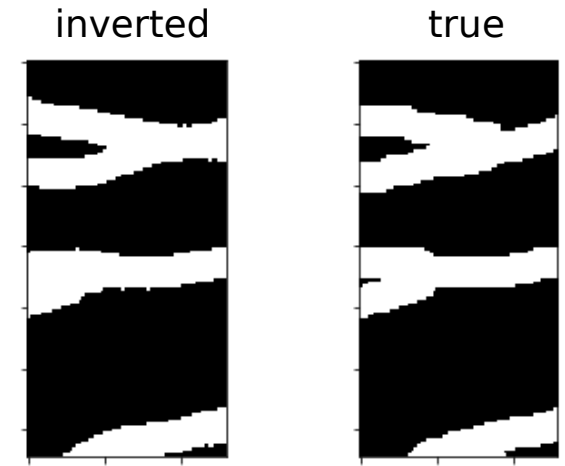
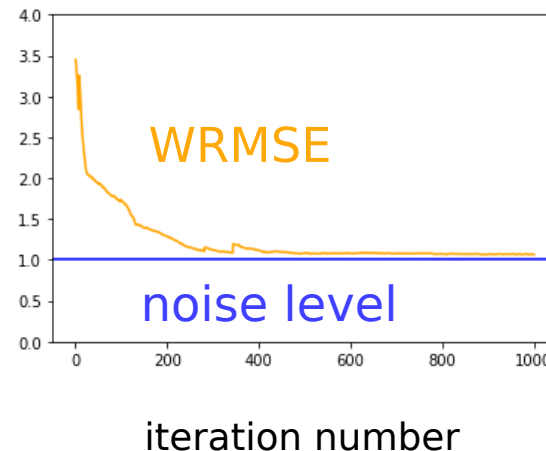
With a Deep Generative Model:

Instead of optimizing w.r.t. (gridded) parameters, do it w.r.t. latent space of DGM



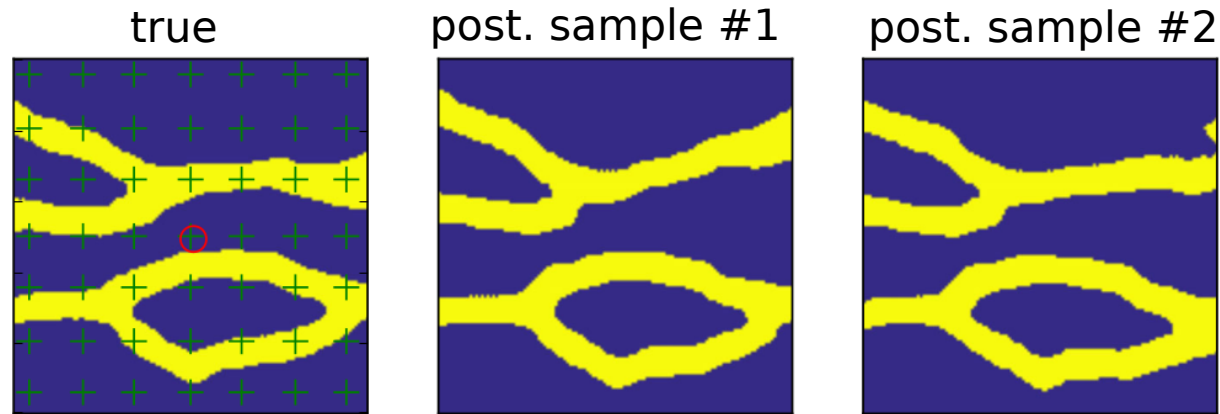
Chain rule:

$$\frac{dS}{d\mathbf{m}} \frac{d\mathbf{m}}{d\mathbf{z}} = \frac{dS}{d\mathbf{z}}$$



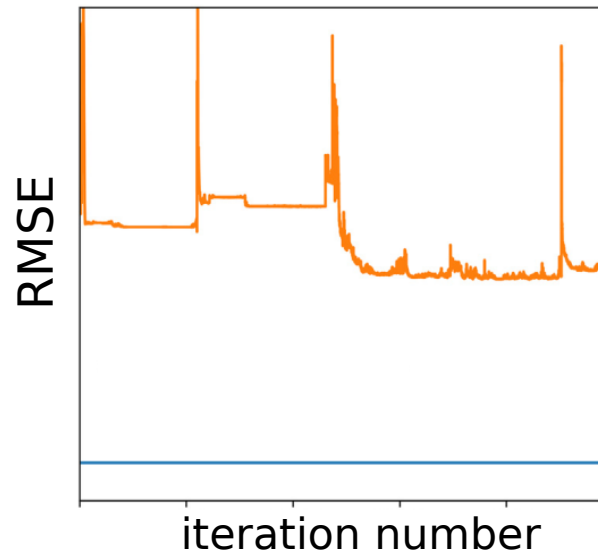
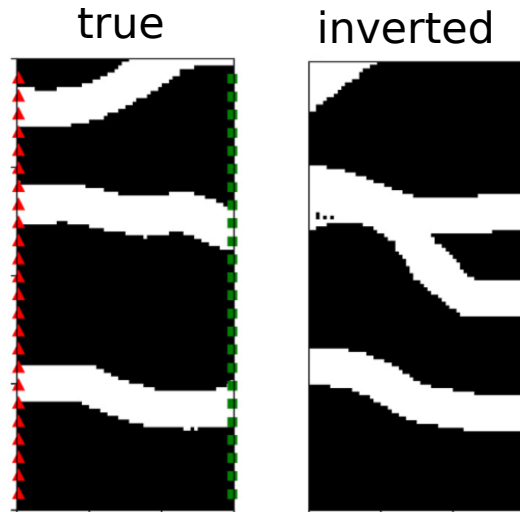
# Generative Adversarial Network (GAN)

Global optimization - MCMC is working fine.



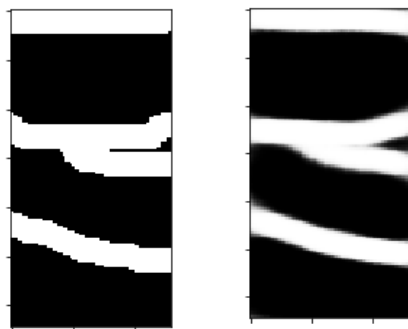
*Laloy et al. 2018, Water Resources Research*

Gradient-based inversion - issues with convergence to local minima.

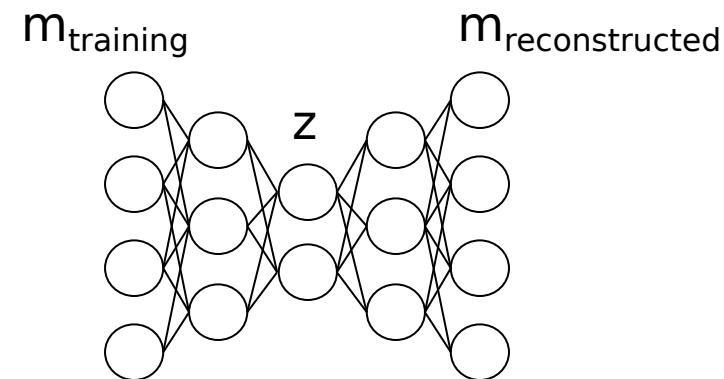


*Laloy et al. 2019, Computers and Geosciences*

## Variational Autoencoder (VAE)

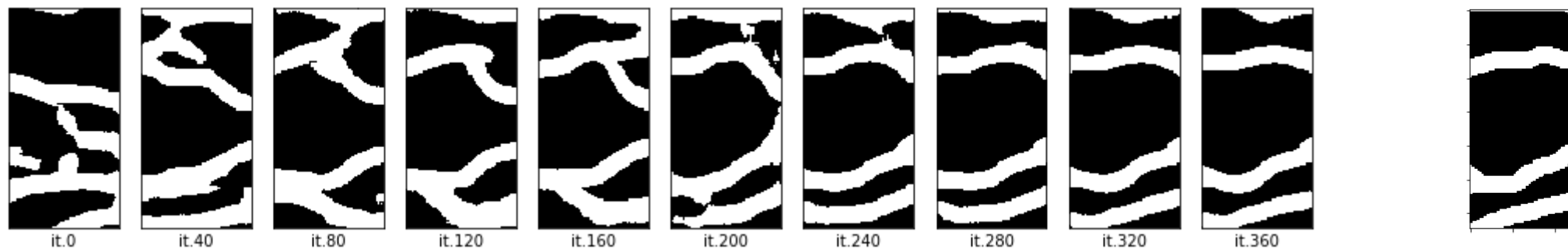


reconstruction  
(not inversion)



A less strict latent space allows easier optimization?

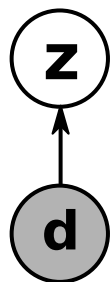
The ability to break up channels



A tradeoff between the fidelity of samples and the easier optimization.

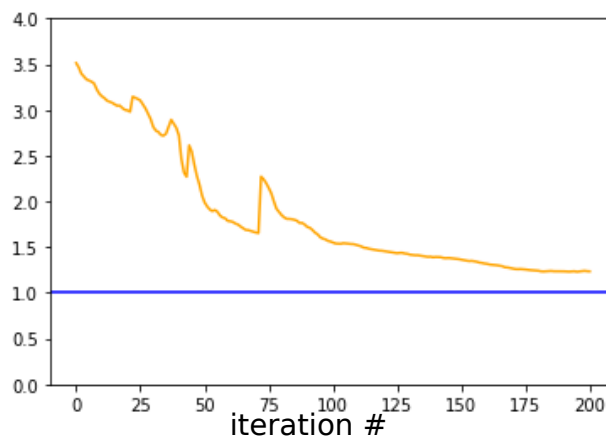
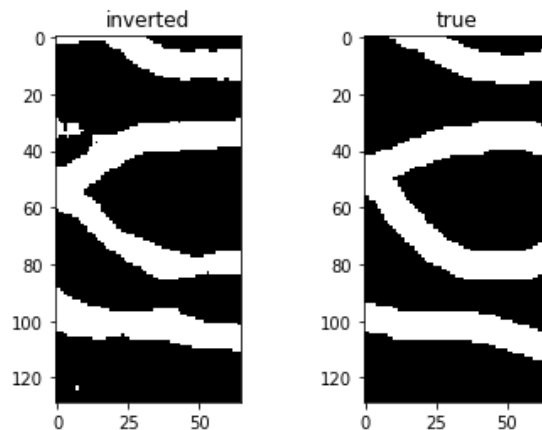
## Comparison of DGMs

Inference (gradient-based inversion)



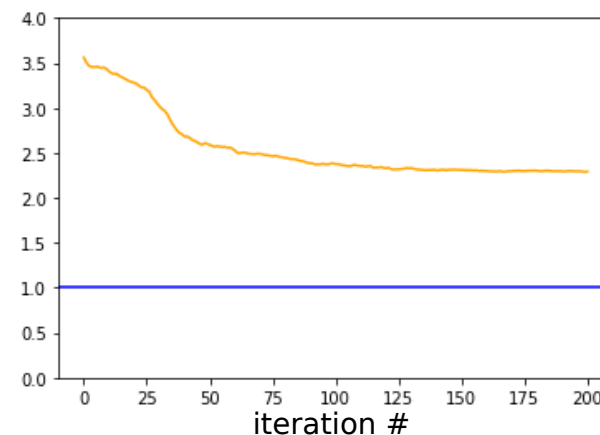
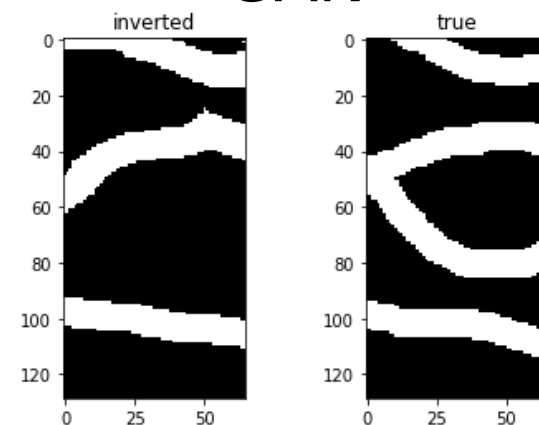
VAE induces a latent space which appears to allow for easier optimization.

VAE



iteration # 0 50 100 150 200

GAN



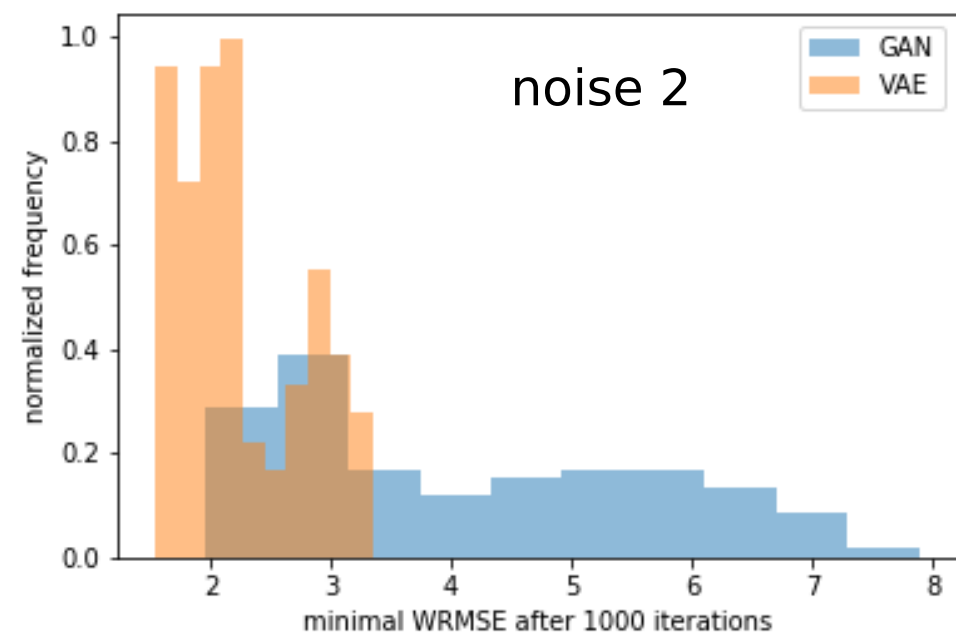
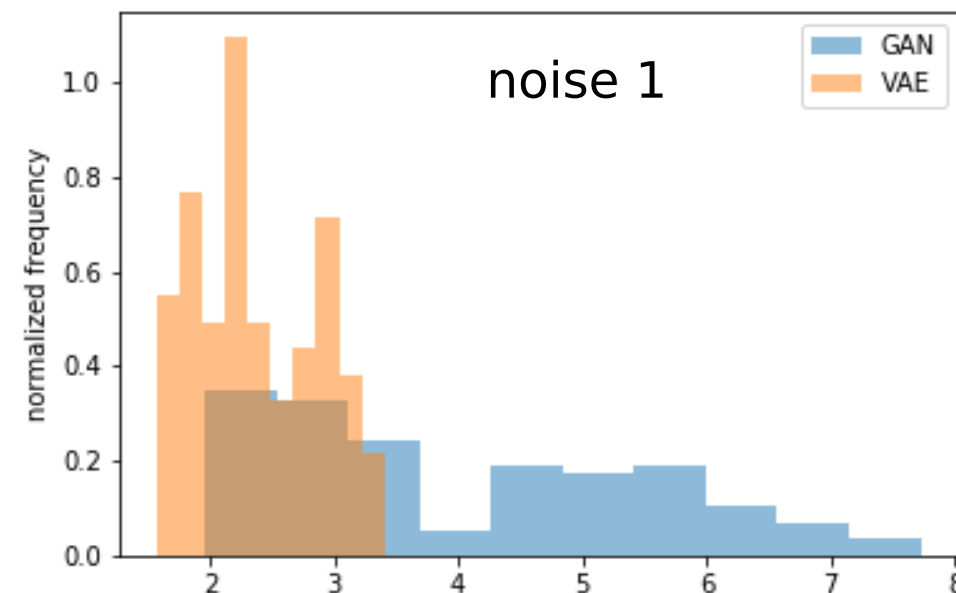
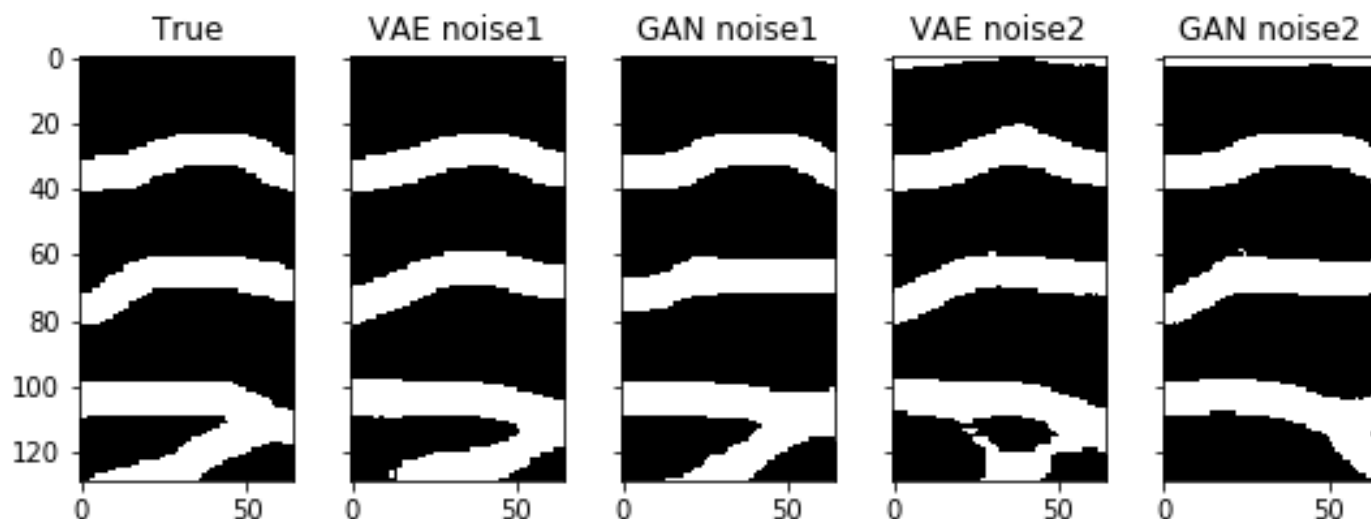
0 50 100 150 200



## Comparison of DGMs

WRMSE comparison for gradient-based inversion  
with GAN and VAE (100 different initial models)

models corresponding to the lowest WRMSE  
(among the 100 tries):



## **Concluding remarks and future work**

VAE latent space is easier to handle with gradient-based optimization compared to GAN latent space.

There is a tradeoff between pattern fidelity and easier optimization.

Is the method still useful for nonlinear forward operators (e.g. shortest path method)?

What is the impact of the VAE model error on the final inversion result?

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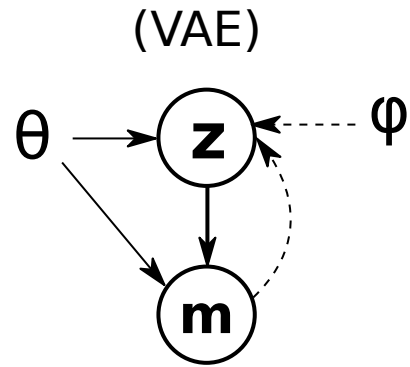
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# VAE theory

Variational Autoencoder



$p(\mathbf{z})$  chosen Gaussian

$p(\mathbf{m}|\mathbf{z})$  obtained from passing  $p(\mathbf{z})$  through a DNN

Since  $p(\mathbf{z}|\mathbf{m})$  is intractable, a variational  $q(\mathbf{z}|\mathbf{m})$  is used whose mean is a DNN too

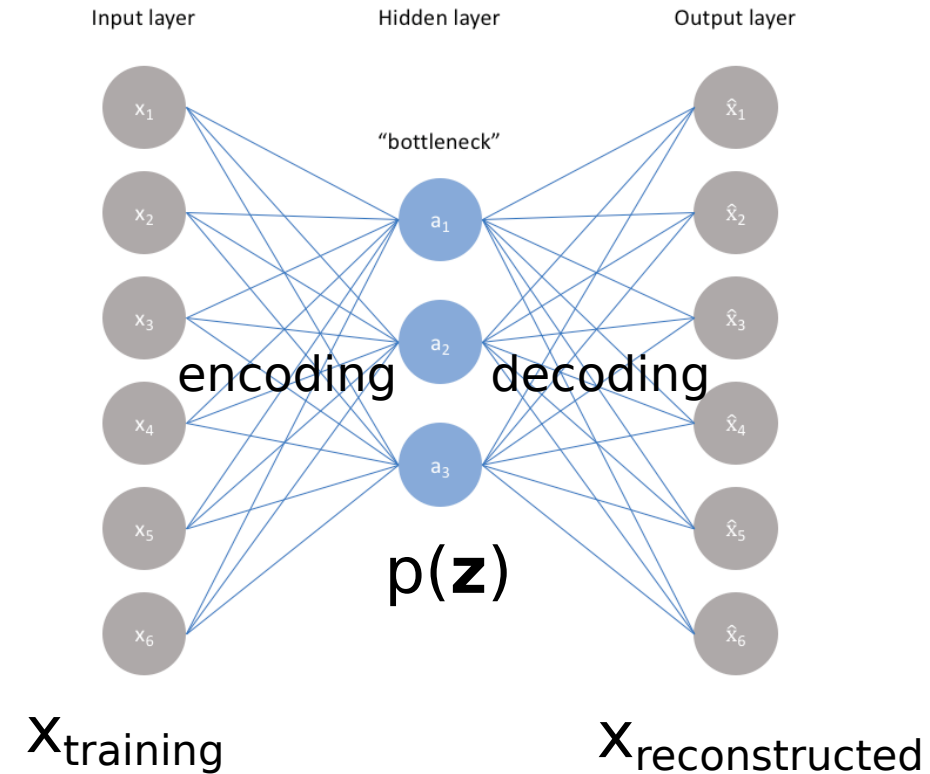
$$p(z) \equiv \mathcal{N}(0, I)$$

$$p(x|z) \equiv \mathcal{N}(f(z), cI)$$

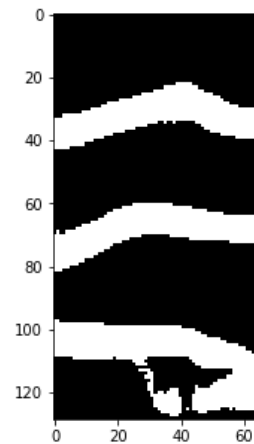
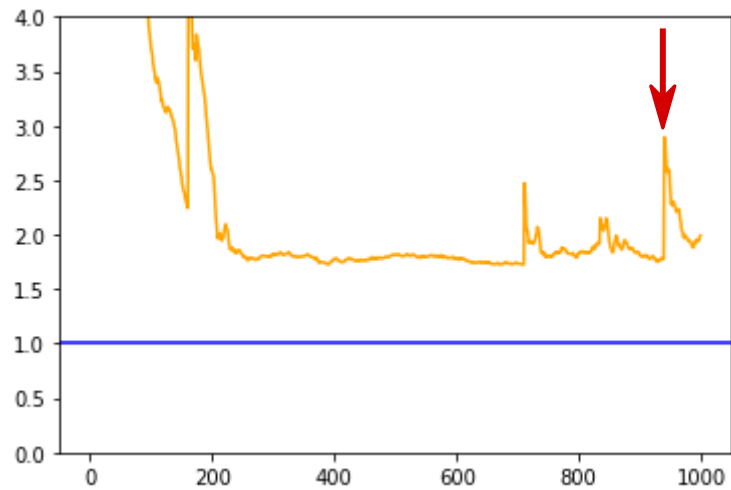
$$q_x(z) \equiv \mathcal{N}(g(x), h(x))$$

Maximize:

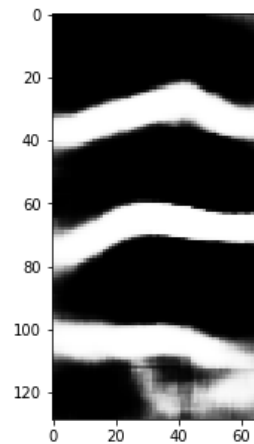
$$\mathcal{L}(\theta, \phi; \mathbf{x}^{(i)}) = -D_{KL}(q_{\phi}(\mathbf{z}|\mathbf{x}^{(i)})||p_{\theta}(\mathbf{z})) + \mathbb{E}_{q_{\phi}(\mathbf{z}|\mathbf{x}^{(i)})} \left[ \log p_{\theta}(\mathbf{x}^{(i)}|\mathbf{z}) \right]$$



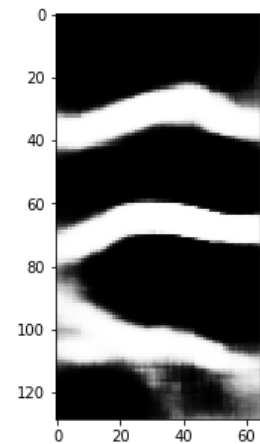
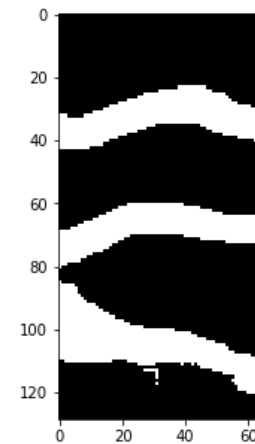
# VAE jumps



it. 939



it. 940



it.935



it.936



it.937



it.938



it.939



it.940



it.941



it.942



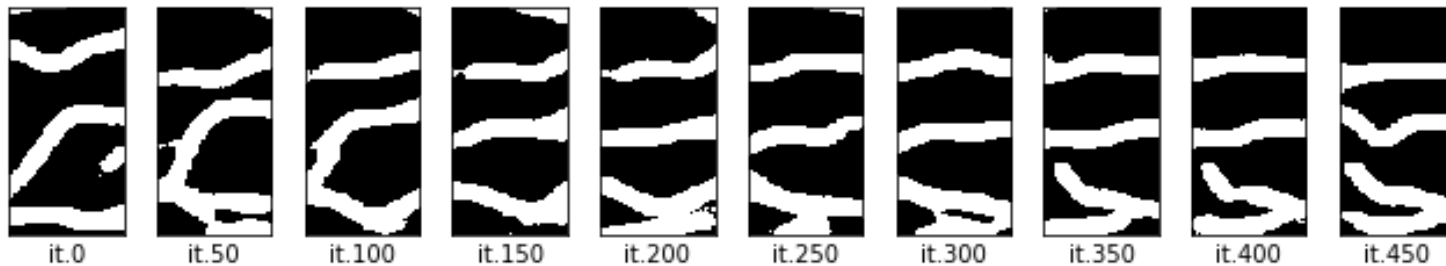
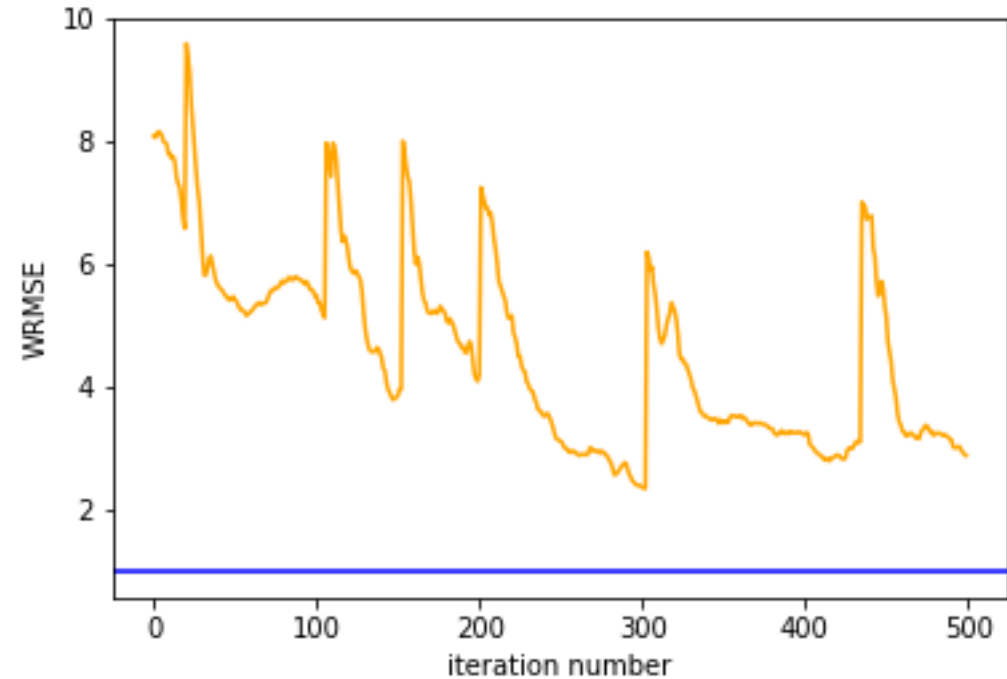
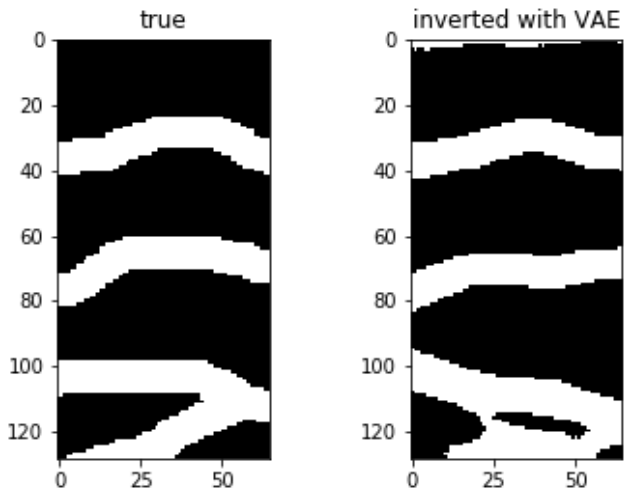
it.943



it.944

# Nonlinear forward traveltime

Does this work with a nonlinear forward model?



# Linear vs nonlinear forward traveltimes

